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## 1. INTRODUCTION

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graph TD; Dynamics[Dynamics] --- Radiation[Radiation]; Dynamics --- StratiformClouds[Stratiform Clouds]; Dynamics --- BoundaryLayer[Boundary Layer]; Dynamics --- CumulusConvection[Cumulus Convection];
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The diagram illustrates the relationship between Dynamics and four other atmospheric processes. Dynamics is positioned at the top, connected by lines to Radiation, Stratiform Clouds, Boundary Layer, and Cumulus Convection, which are arranged horizontally below it.

For example, in the stratocumulus-to-cumulus transition region, stratocumulus clouds break up into a combination of shallow cumulus and broken stratocumulus. Shallow cumulus clouds may be considered to reside completely within the PBL, or they may be regarded as starting in the PBL but terminating above it. Deeper cumulus clouds often originate within the PBL but also can originate aloft. To the extent that our models separately parameterize physical processes which interact strongly on small space and time scales, the currently fashionable practice of modularization may be doing more harm than good.

A key ingredient of any modern cumulus parameterization is a cloud model, which describes how convective clouds interact with their environment. The cloud model provides answers to such questions as:

- What sorts of clouds will grow in this sounding?
- Through what mechanisms do the clouds alter the sounding?

A cumulus parameterization also needs a closure, which essentially determines how fast the convective engine is running, as measured by the precipitation rate or the convective mass flux.

The cloud model determines the in-cloud sounding, including the cloud base and cloud top levels. Some parameterizations, notably that of Arakawa and Schubert (1974; hereafter AS), permit multiple cloud "types" to coexist in a given grid column. AS assumed that all clouds originate at the top of the PBL, and they permitted one cloud type for each model level above the PBL top. AS were motivated in part by the observed coexistence of deep and shallow clouds. More recently Johnson et al. (1996) have reported observations of a third class of tropical cumuli, which characteristically detrain near the freezing level.

Ding and Randall (1998) have generalized the spectral cloud model of AS to permit multiple simultaneous cloud bases, as well as multiple simultaneous cloud tops for each cloud base. Although convection does sometimes originate above the PBL, this is rarely discussed in the literature. In the parameterization of Ding and Randall (1998), the number of cloud types is thus proportional to the square of the number of model layers, which is worrisome in this era of rapidly increasing vertical resolution.

### 3. CUMULUS TRANSPORTS

The term "convection" denotes transport, rather than latent heat release or precipitation. When we speak of "cumulus convection," we are therefore implicitly assigning high importance to the very strong vertical transports associated with the convective circulations. Convective fluxes are associated with a convective mass flux:

$$\begin{aligned}\rho \overline{w'h'} &= \iint_{hw} \rho p(w, h) (w - \bar{w})(h - \bar{h}) dw dh \\ &= \int_{\lambda} \rho \sigma(\lambda) [w_c(\lambda) - \bar{w}] [h_c(\lambda) - \bar{h}] d\lambda\end{aligned}\quad (1)$$

Here  $\rho$  is density, and  $w$  is vertical velocity. On the first line of this equation, we represent the convective flux of a quantity  $h$ , which might represent the moist static energy, as the integral of the joint probability density function (PDF) denoted by  $p(w, h)$ , multiplied by the departures of the vertical velocity and moist static energy from their grid-cell averaged values at each level. On the second line, we follow AS and rewrite this flux in terms of an integral over cloud type, where the cloud type is denoted by the independent variable  $\lambda$ , of the product of a convective mass flux,  $M_c(\lambda) \equiv \rho \sigma(\lambda) [w_c(\lambda) - \bar{w}]$ , with the cloud-environment moist static energy difference. Here  $\sigma(\lambda)$  is the fractional area covered by each cloud type. On the first line, the cloud model is represented through the PDF, while on the second it is represented through the mass flux and the in-cloud moist static energy. This makes the point that *we can think of the cloud model as a joint PDF* involving vertical velocity and various

thermodynamical and/or dynamical quantities.<sup>1</sup> Conversely, we can think of the AS cloud model, with its cloud-type parameter,  $\lambda$ , as a clever device to generate PDFs for a given sounding.

Consider the application of the AS parameterization (with a single cloud-base level at the PBL top) to a vertically discrete model with  $N$  layers, such that the lowest layer is the PBL. There are then  $N - 1$  cloud types (some or all of which may not actually occur on a given time step in a given grid column). At a given model level, say in the middle troposphere, a certain number of cloud types will penetrate, as they cloudy air rises from the PBL to the ultimate cloud top height. (Other clouds may stop below the level in question.) This situation is illustrated in Fig. 2. The black dots represent convective updrafts, while the "x" represents a

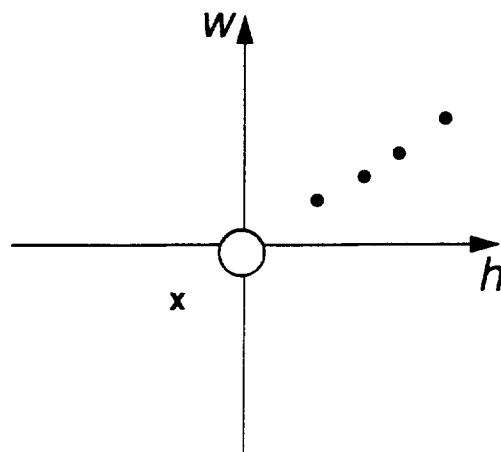


Figure 2: A schematic illustration of a PDF for moist static energy and vertical velocity. The large grey circle near the origin represents the environment. The black dots represent convective updrafts penetrating through a given level. The x represents a convective downdraft.

convective downdraft. The large grey blob represents the environment.<sup>2</sup> In this example, the PDF at the level in question is represented by six delta functions (four for the updrafts, one for the downdraft, and one for the environment).

Now suppose that the number of layers in the numerical model is doubled, while the meteorological situation remains unchanged. Because the number of model layers doubles, the number of cloud types can be expected to roughly double as well. In particular, the number of cloud types penetrating through the level in question can be expected to double. This means that *the complexity of the PDF at a given level depends on the vertical resolution of the model* -- an undesirable property of the cloud model, in our opinion. While this problem occurs with the AS scheme, it is even more troublesome in the multiple-cloud-base scheme of Ding and

<sup>1</sup> Because the PDF extends over all of the air of a grid cell, it actually describes the environment as well as the clouds.

<sup>2</sup> Here the relatively large size of the environmental blob is simply meant to convey that the environment occupies a very large fraction of the grid cell; in reality the quiet environment would be represented by a very narrow spike in the PDF, while the turbulent clouds might be represented by somewhat broader spikes.

Randall (1998). As a goal for the future, it would be useful to develop a cloud model which corresponds to a PDF whose complexity at each level is independent of the vertical resolution of the model.

Fig. 3 shows an example of  $p(w, h)$  as simulated by the cloud model of S. Krueger of the University of Utah, plotted for three different levels. Note that the mean of  $h$  first decreases upward, then increases upward, as would be expected for a deep convective sounding. The PDFs in Fig. 3 do not closely resemble the sketch in Fig. 2, suggesting that realistic cloud models depart significantly from the idealized picture of a collection of entraining plumes. On the other hand, we should remember that the cloud model contains a lot of "noise," including that associated with gravity waves and turbulent stratiform clouds, at least some of which is probably quite irrelevant to net vertical transports.

#### 4. PROGNOSTIC CUMULUS PARAMETERIZATIONS

Until recently, all cumulus parameterizations have been purely diagnostic in the sense that they did not introduce any prognostic variables. Randall and Pan (1993) and Pan and Randall (1998) introduced a prognostic parameterization, in which the prognostic variable is the vertically integrated cumulus kinetic energy,  $K$ , associated with each cloud type, which satisfies an equation of the form

$$\frac{\partial K}{\partial t} = M_B A - \frac{K}{\tau_D}. \quad (2)$$

Here  $M_B$  is the cloud-base mass flux for the cloud-type in question,  $A$  is the cloud work function for that cloud type, and  $\tau_D$  is a dissipation time scale. The cloud-base mass flux is assumed to be related to the cumulus kinetic energy by

$$K = \alpha M_B^2, \quad (3)$$

where  $\alpha$  is a dimensional parameter which is currently prescribed. See Pan and Randall (1998) for more discussion. Note that  $K$  represents the vertically integrated convective kinetic energy in not only the vertical motion, but also the horizontal parts of the convective circulations. Therefore,  $K$  is not all that similar to the familiar "turbulence kinetic energy" used in many PBL parameterizations.

Nevertheless, there is an unmistakable similarity between Eq. (2) and the turbulence kinetic energy equation which is used in higher-order closure (HOC) models. The point is that cumulus parameterizations are, perhaps, evolving to become more similar to the parameterizations used to represent the turbulence in the PBL.

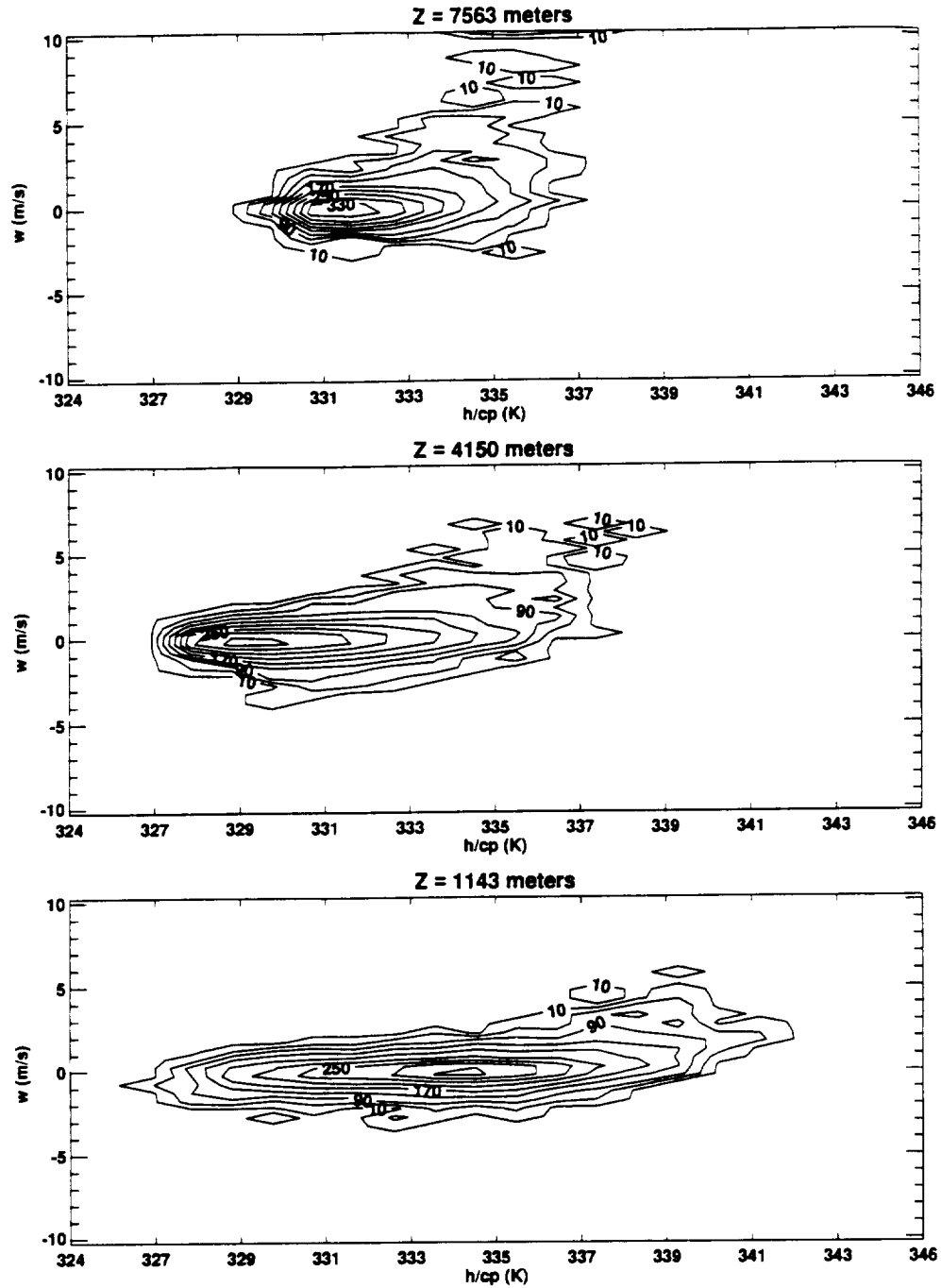


Figure 3: The logarithm of the joint PDFs of moist static energy (horizontal axis) and vertical velocity (vertical axis) as simulated with the cloud model of S. Krueger, for a deep convective case. The PDFs are shown for three levels, as indicated. The units are arbitrary.

## 5. MASS FLUX METHODS IN PBL PARAMETERIZATIONS

It is also true that PBL parameterizations are now making use of mass flux closures (MFCs), analogous to those which are used in cumulus parameterizations. Examples appear in the work of Wang and Albrecht (1986), Chatfield and Brost (1987), Penc and Albrecht (1987), Randall et al. (1992), and Lappen (1999), among others.

Lappen (1999) developed a model of PBL turbulence and shallow cumulus convection, which combines HOC and MFC. HOC models must include closures for higher-order moments (e.g., fourth moments in third-order closure models), for pressure terms, and for dissipation terms. Mass-flux closure (MFC) models have been proposed for parameterization of cumulus convection and, more recently, the convective PBL. MFC models include closures for lateral mass exchanges, and pressure terms (which are usually ignored).

Lappen's model makes use of a simple assumed joint PDF for the variables of interest. The PDF consists of two delta-functions, one for the updrafts and one for the downdrafts. Lappen shows that the equations typically used in HOC models can be derived by integrating over the PDF. Accordingly, the model is called "Assumed Distribution Higher-Order Closure" (ADHOC). The prognostic variables of ADHOC are the mean state, the second and third moments of the vertical velocity, and the vertical fluxes of other quantities of interest. Lappen shows that all of the parameters of the distribution can be determined from the predicted moments; thereafter the joint distribution is effectively known, and so any and all moments can be constructed as needed. In this way, the usual closure problem of "higher moments" is avoided. The pressure-term parameterizations previously developed for HOC models are used to predict the convective fluxes and the moments of the vertical velocity. ADHOC includes a new parameterization of the "entrainment" and "detrainment" which lead to mass exchanges between vertical currents, and also a parameterization of "sub-plume-scale" fluxes and their effects on both the mean state and the larger eddies. ADHOC may be particularly well suited to the simulation of regimes in which the PBL turbulence and the cumulus convection are not well separated, e.g. the broken stratocumulus and shallow cumulus regimes. Lappen (1999) presents numerical results of such simulations, which are quite encouraging.

To indicate the potential of ADHOC to represent both PBL and cumulus processes, consider the ADHOC form of the prognostic equation for the variance of a scalar variable  $h$ , which might be moist static energy:

$$\begin{aligned}
 & \frac{\partial}{\partial t} m \sigma (1 - \sigma) (h_{\text{up}} - h_{\text{dn}})^2 \\
 & = -(E + D) (h_{\text{up}} - h_{\text{dn}})^2 \\
 & - \frac{\partial}{\partial z} [M_c (1 - 2\sigma) (h_{\text{up}} - h_{\text{dn}})^2] \\
 & - M_c (h_{\text{up}} - h_{\text{dn}}) \frac{\partial}{\partial z} \bar{h} .
 \end{aligned} \tag{4}$$

Here the subscripts “up” and “down” denote the updrafts and downdrafts, respectively;  $\sigma$  is the fractional area covered by the updrafts;  $m$  is the density;  $E$  and  $D$  are the entrainment and detrainment mass fluxes, respectively (here entrainment denotes flow from the downdraft to the updraft);  $M_c$  is the convective mass flux. Neglecting the tendency term, we can show (see also Randall et al. 1992) that for  $\sigma = 1/2$  (4) reduces to

$$\overline{mw'h'} = \frac{-2M_c^2}{E+D} \frac{\partial \bar{h}}{\partial z}. \quad (5)$$

This describes down-gradient diffusion in which the effective eddy diffusivity is represented by

$$K_{eff} = \frac{2M_c^2}{E+D}. \quad (6)$$

Another limiting case is that of deep cumulus convection, for which  $\sigma \ll 1$ . For this case, we can show that (4) reduces to

$$\frac{\partial}{\partial z} \overline{mw'h'} = -M_c \frac{\partial \bar{h}}{\partial z} - D(h_{up} - h_{down}), \quad (7)$$

which is the famous “compensating subsidence” formula derived by AS. ADHOC is thus able to represent both downgradient diffusion and penetrative convection in a single framework.

Fig. 4 shows a comparison of the ADHOC-simulated virtual static energy flux, vertical velocity variance, and vertical velocity skewness with aircraft observations analyzed by de Laat and Duijnkerke (1998) for ASTEX. ASTEX collected data in a “transitional” regime midway between a classical stratocumulus (Sc) regime (e.g., coastal California), and a trade-wind (TW) cumulus regime (e.g., BOMEX). In Fig. 4, we see that ADHOC does a good job of reproducing the observations and is able to capture the transitional nature of ASTEX. For example, the profile of  $\overline{w'w'}$  in a TW regime has two distinct maxima, with a significant minimum in between. In a “classic” Sc regime, however, there is only one peak in  $\overline{w'w'}$ . In Fig. 4, both ADHOC and the observations show a double peak, but the minimum separating the peaks is very weak relative to that observed in the trades. Similarly, the profile of  $\overline{w's'}$ , simulated by ADHOC has some TW characteristics in that the surface and the cloud layer are almost decoupled, but the magnitude of the decoupling is weak compared with what is observed in the trades. The simulated skewness profile is also physically realistic. Near the surface, the skewness is small and positive, as a result of weak surface fluxes; at cloud top, it is negative, indicating the presence of narrow downdrafts, and near cloud base, the skewness is relatively large and positive, signaling the presence of narrow cumulus updrafts driven by condensational heating.

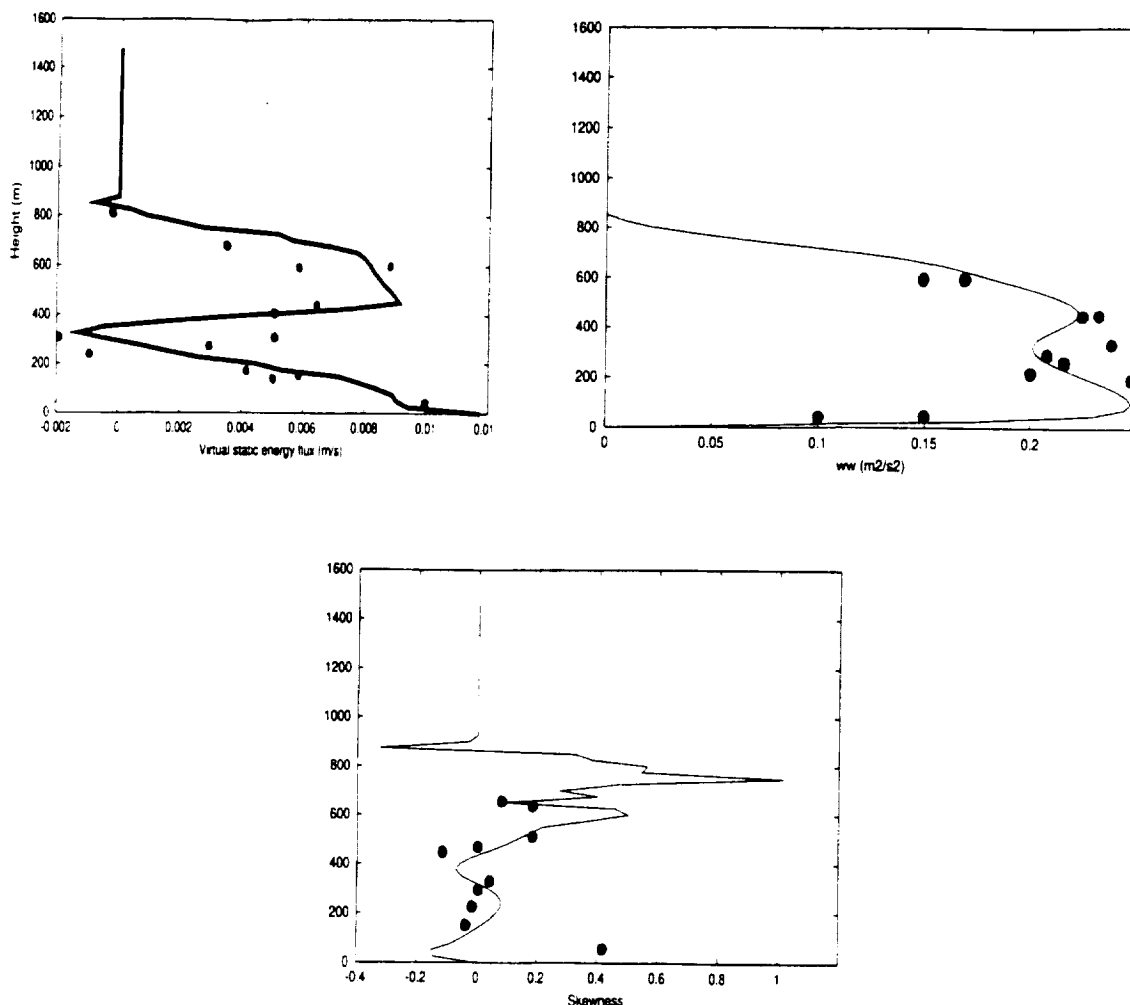


Figure 4: Comparison of the ADHOC-simulated virtual static energy flux (top, left), vertical velocity variance (top, right), and vertical velocity skewness (bottom) with those observed by aircraft. The solid line is ADHOC and the dots are aircraft observations from the NCAR electra as analyzed by de Roode and Duynkerke (1997).

## 6. CONCLUSIONS AND A DREAM

We have argued that parameterizations of cumulus convection make use of PDFs which can be viewed as “cloud models.” Prognostic cumulus parameterizations are now emerging, which are analogous to the HOC models used in PBL parameterizations. At the same time, PBL parameterizations are making use of mass-flux methods which were originally developed for cumulus parameterizations, and these are being combined with HOC methods based on assumed PDFs, as in ADHOC.

These trends suggest that cumulus parameterizations and PBL parameterizations are undergoing a kind of convergent evolution. To the extent that cumulus processes and PBL processes are intimately coupled, as in the case of shallow cumulus convection which is



occurring over roughly half the Earth's surface at any given time, it would be very nice to represent the effects of both processes with a single physical parameterization and in a unified computational framework.

Such a goal calls into question the idea that our models can and should be highly modular, e.g. with cumulus convection modules and PBL modules. Nature is not modular. There can be some short-term conceptual and practical benefits from creating modules in our models, but in the longer term our models should not be more modular than the natural processes which they are intended to represent.

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